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Topological Indices and Connectivity Indices of Delhi Metro Rail Corporation

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Abstract: New Delhi is the capital of India and a highly dense area in India. Many people travel from their homes to their corresponding offices, colleges, and factories. A well-connected public transport network is required for this purpose. The well-connected metro rail network is significant for economic growth and public transport. From time to time, expand this network for future traffic load. In this paper, we calculate several topological and connectivity indices, graph energy for the DMRC (Delhi Metro Rail Corporation) phase I and phase II, to understand how these quantities can improve the DMRC network and find a connection between them, if any.

Keywords: *Topological Indices, Zagreb Indices, Connectivity Indices Alpha, Beta, and Gamma, Graph Energy, DMRC.*

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I. INTRODUCTION

A. Delhi Metro Rail Corporation:

Delhi's population is now estimated at 34.7 million in 2025 [1]. It is the 2nd most populous city in the world. For the smooth convenience of transport, Delhi requires a well-connected public transport network. For this purpose, the DMRC was registered on May 3, 1995, under the Companies Act, 1956. It is a joint venture between the Central Government and the Government of the National Capital Territory of Delhi (GNCTD), with 50:50 equity participation. Its purpose is to implement the construction and operation of a world-class Mass Rapid Transport System (MRTS) in Delhi. [2]

Construction of Delhi Metro phase I, with 65.1 km of routes, began in 1998. The first section opened in 2002, and the last section opened in 2006. The estimated cost of phase I is 11252 Cr. Rupees.

➢ *Funding Plan of Phase I*

Funding Plan of Phase-I (including Dwarka Sub-City)

Particulars	Amount (Rs./Crore)	%
JICA Loan	6,356.45	56.49%
GOI Equity	1,464.00	13.01%
GNCTD Equity	1,464.00	13.01%
Subordinate debt for Land	865.00	7.69%
Grant	320.00	2.84%
Property Development	782.55	6.96%
Total	**11,252.00**	**100%**

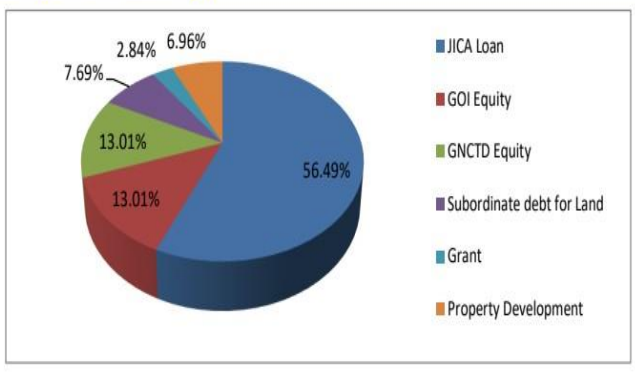

Fig 1 [3] Funding Plan of Phase I

- This initial phase contains 59 stations and three metro lines: the red, yellow, and blue lines.

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➢ *Map of Phase I with Station Name*

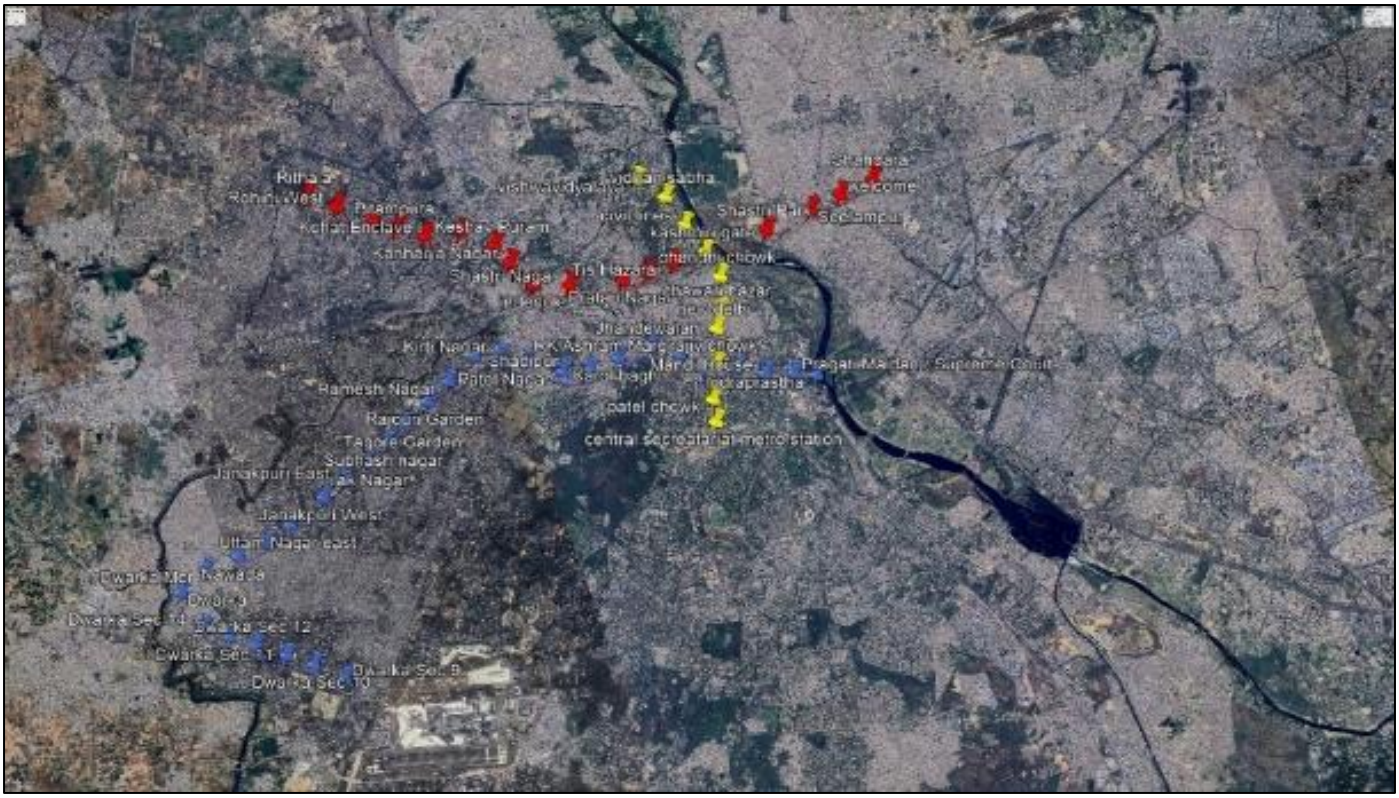

Fig 2 Map of Phase I with Station Name
[Own Work]

➢ *Map of Phase I without Station Name*

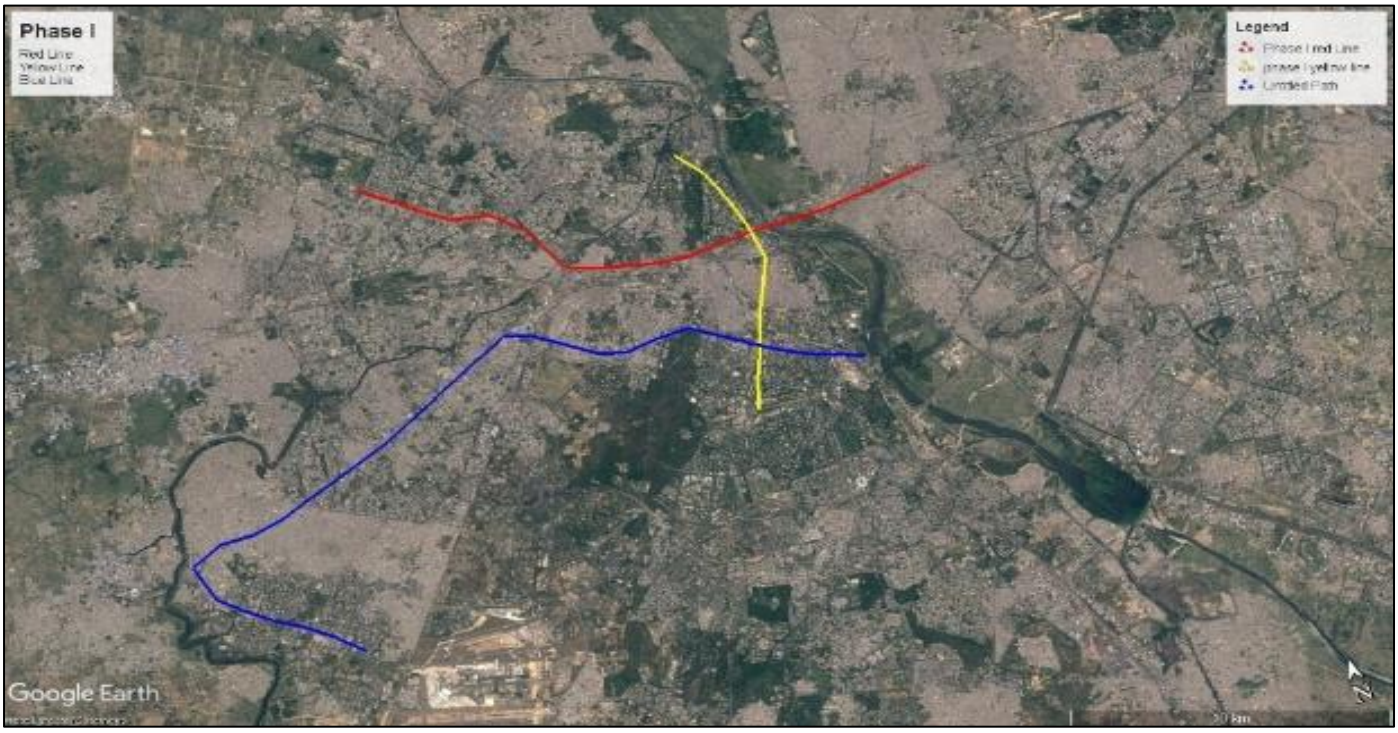

Fig 3 Map of Phase I without Station Name
[Own Work]

- Delhi Metro Phase II length of 124.93km, began in 2006. The estimated cost of phase II is 21787.27Cr. Rupees.

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➢ *Funding Plan of Phase II*

Funding Plan of Phase-II (including NCR and Dwarka)

Particulars	Amount (Rs./Crore)	%
JICA Loan	10,231.62	46.96%
GOI Equity	3,078.70	14.13%
GNCTD Equity	3,078.70	14.13%
Subordinate debt for Central Taxes	638.50	2.93%
Subordinate debt/Grant for State Taxes	643.70	2.95%
Subordinate debt for Land	848.00	3.89%
Land free of Cost	52.00	0.24%
Grant	1,833.94	8.42%
DMRC Internal Accruals/Property Development	1,382.11	6.34%
Total	**21,787.27**	**100%**

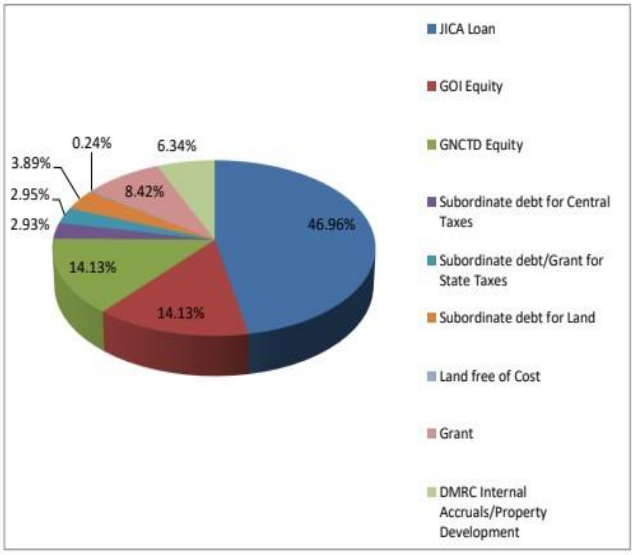

Fig 4 Funding Plan of Phase II

Phase II contains 86 stations and introduces three new lines: green, orange, and purple lines, as well as the extension of the previous three lines of Phase I.

➢ *Map of Phase II with Station Name*

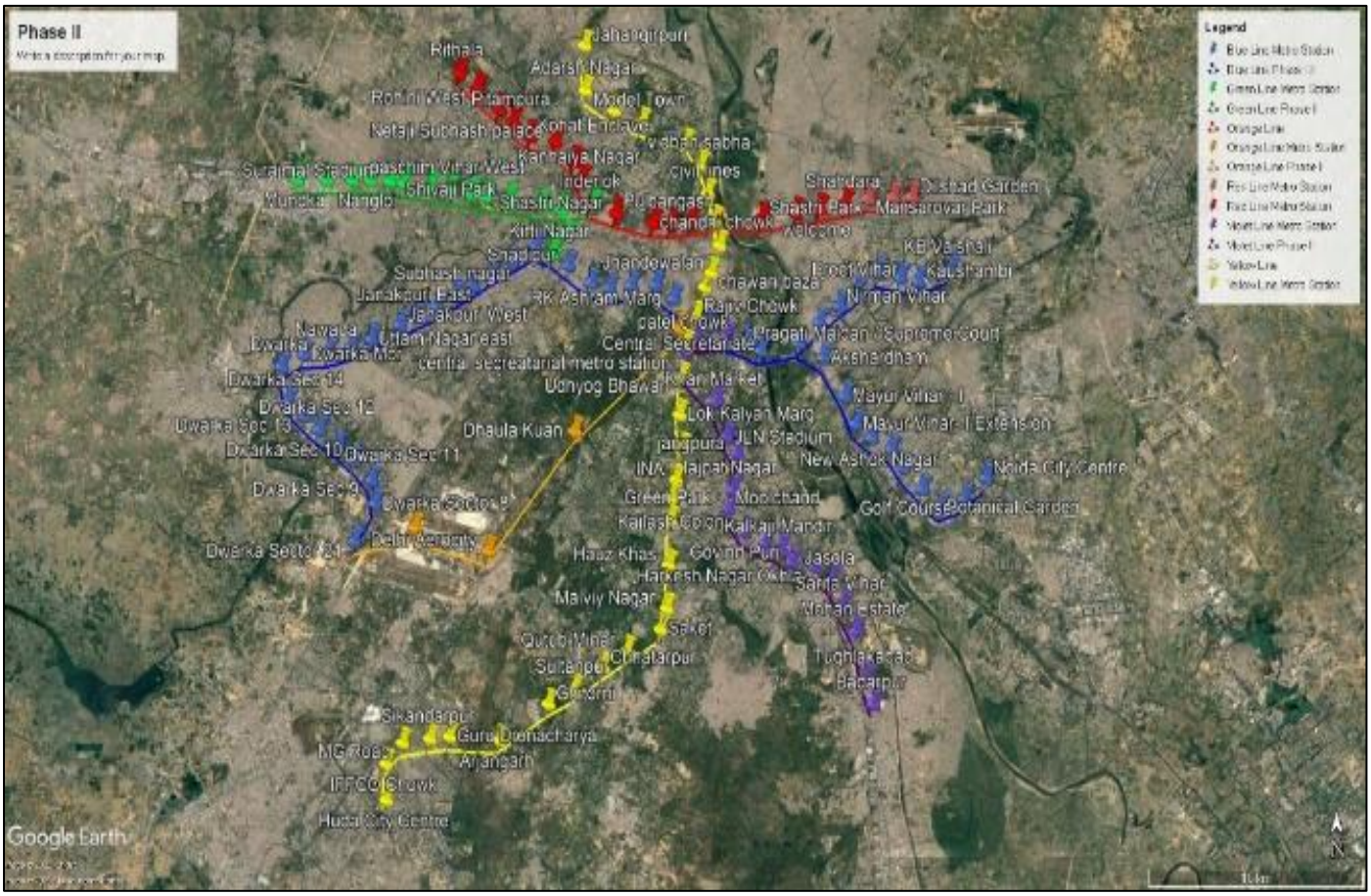

Fig 5 Map of Phase II with Station Name
[Own Work]

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➢ *Map of Phase II without Station Name*

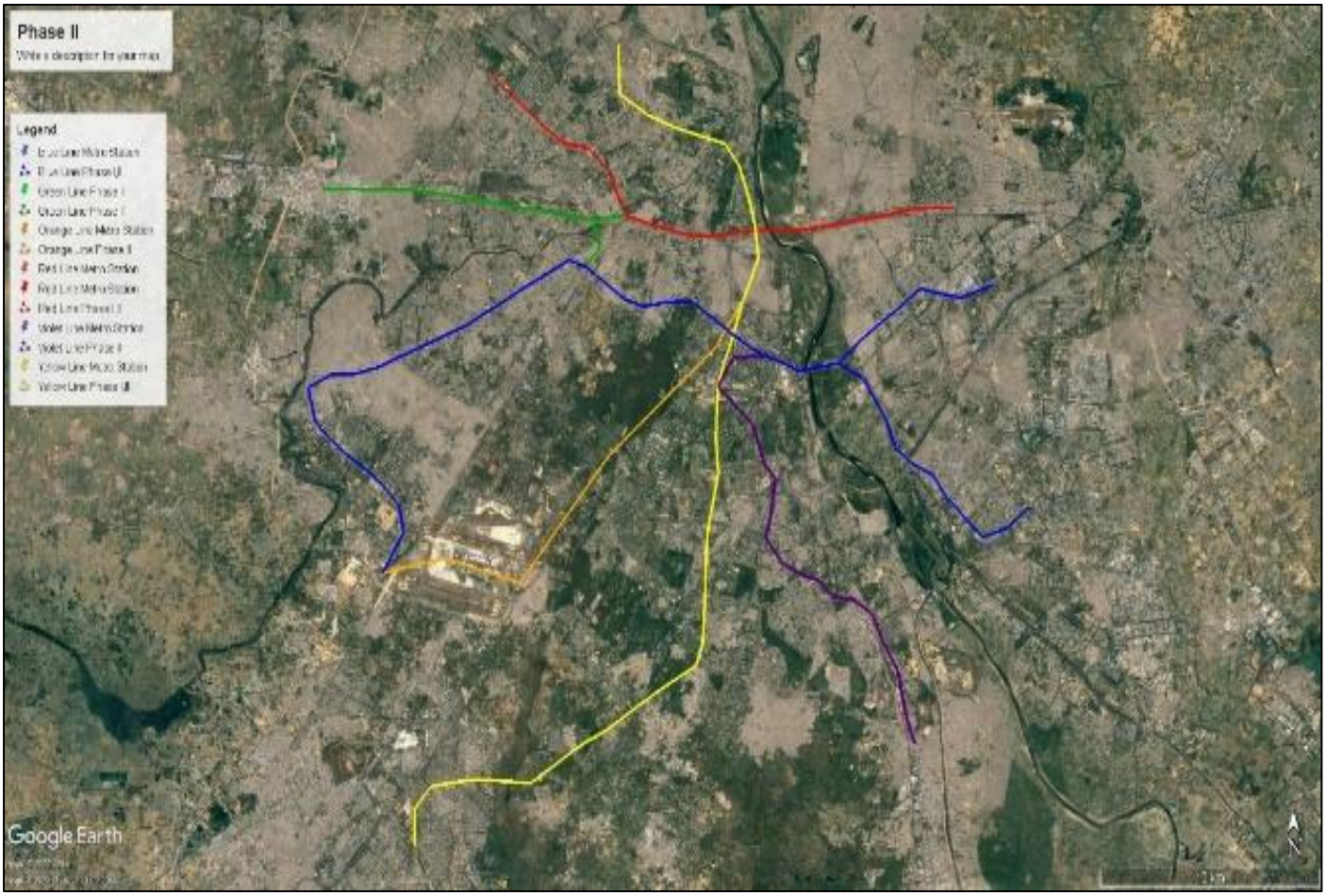

Fig 6 Map of Phase II without Station Name
[Own Work]

- To do the work, we require graph theory.

B. Graph Theory:

- Graph theory is a branch of mathematics. A graph G is a triple consisting of a vertex set V(G), an edge set E(G), and a relation that associates with each edge two vertices called its endpoints [11]. Graph theory has numerous applications in chemical graph theory, specifically in QSAR (Quantitative Structure-Activity Relationship) and QSPR (Quantitative Structure-Property Relationship), which are useful in medicinal chemistry and pharmaceutical science. Graph theory has two branches: Spectral graph theory and Chemical graph theory.
- Topological indices are the connector between spectral graph theory and chemical graph theory.
- Topological indices are mainly of two types: degree-based and distance-based topological indices. Some degree-based topological indices are the First Zagreb Indices and the Second Zagreb Indices.

✓ *The First Zagreb Index is Defined as*

$$M_1(\mathrm{G}) = \sum_{u\in \mathrm{V(G)}} {d_u}^2 \quad [1]$$

✓ *The Second Zagreb Index is Defined as*

$$M_2(\mathrm{G}) = \sum_{uv\in \mathrm{E(G)}} d_u d_v \quad [2]$$

Where d_u is the Degree of Vertex u.

- Some other types of indices, which also help us in learning about a structural network, are connectivity indices.

Alpha Index: $\alpha = \frac{e-v+1}{2v-5}$

The alpha index gives the range value from 0 to 1.

Beta Index: $\beta = \frac{e}{v}$

Gamma Index: $\gamma = \frac{e}{3v-6}$

Where e is the number of edges in the graph and v is the number of vertices in the graph.

- Adjacency Matrix: It represents the whole set of connections between adjacent pairs of vertices. The entries a_{ij} of the matrix equal 1 if vertices v_i and v_j are adjacent and zero otherwise [10].

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- Graph Energy: The arithmetic sum of the absolute values of the eigenvalues of the graph's adjacency matrix.[6]

II. RESEARCH METHODOLOGY

➢ *Data Collection:*

The data was collected from the official website of the Delhi Metro and the government of India.

➢ *Image Acquisition:*

We use Google Earth Pro to high resolution and detailed images of Delhi.

➢ *Metro Station Tracing:*

Google Earth Pro was used to trace the metro station and visualize the metro line.

➢ *Phase I :*

- *Path of Phase I*

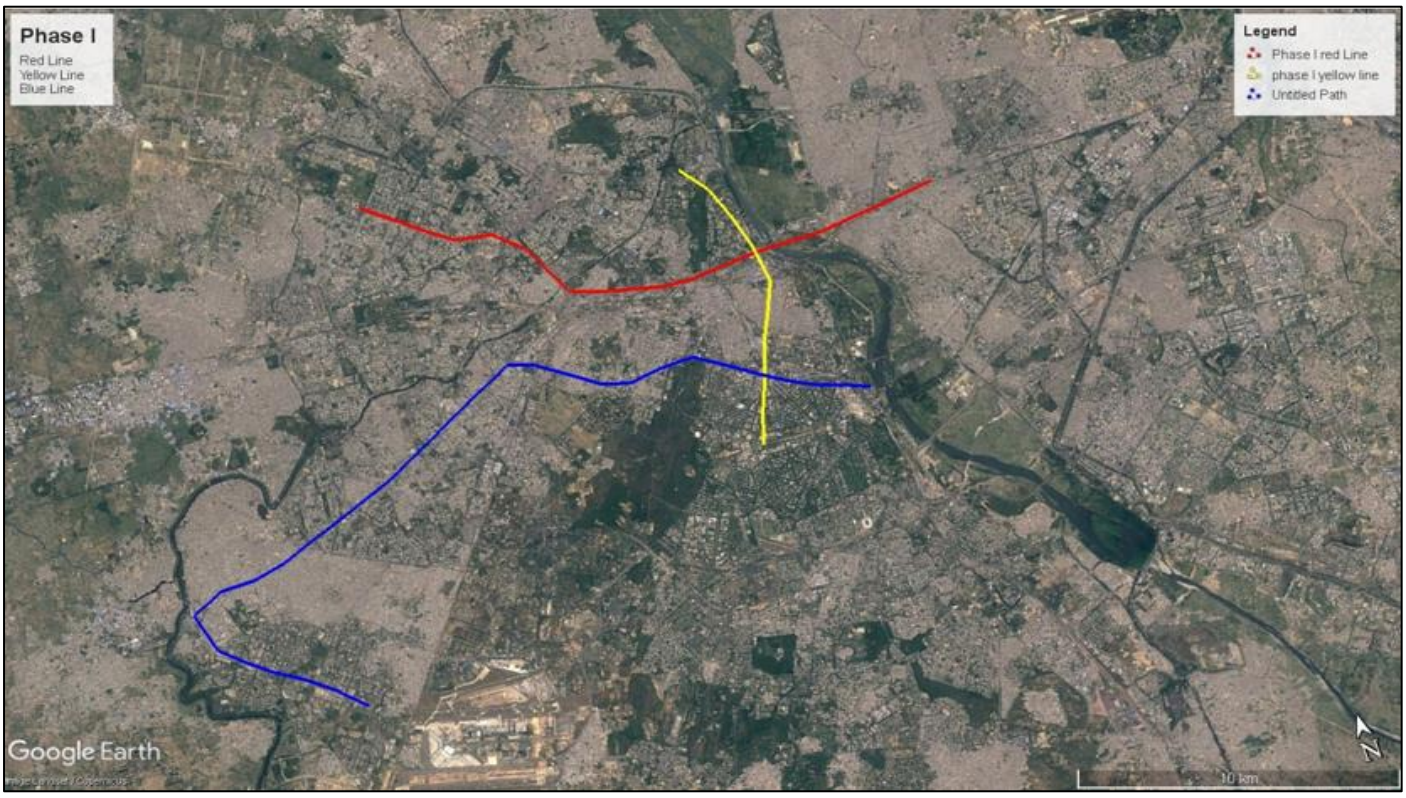

Fig 7 Path of Phase I
[Own Work]

- *Graph of Phase I*

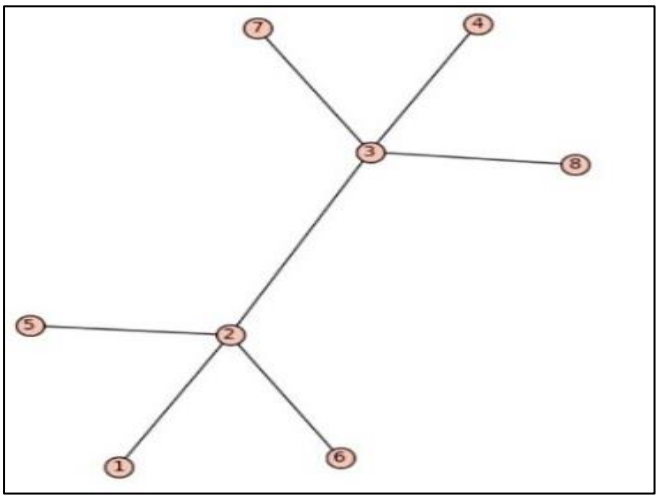

Fig 8 Graph of Phase I
[Own Work]

Only endpoints of metro lines and interchange stations are chosen for nodes/vertices.

For the Phase I graph e = 7, v=8, d̀(1) =1, d̀(2) =4, d̀(3) =4, d̀(4) =1, d̀(5) =1, d̀(6) =1, d̀(7) =1, d̀(8) =1

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Table 1 Calculation for Phase I

Indices	Calculations	Result
$M_1(G)$ First Zagreb Indices	$d^2(1) + d^2(2) + d^2(3) + d^2(4) + d^2(5) + d^2(6) + d^2(7) + d^2(8)$	38
$M_2(G)$ Second Zagreb Indices	$d(1).d(2) + d(2).d(3) + d(2).d(5) + d(2).d(6) + d(3).d(4) + d(3).d(7) + d(3).d(8)$	40
α Alpha Index	$\alpha = \frac{e - v + 1}{2v - 5}$	0
β Beta Index	$\beta = \frac{e}{v}$	0.875
γ Gamma Index	$\gamma = \frac{e}{3v - 6}$	0.38

- *Adjacency Matrix:*

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues of adjacency matrix are 0,0,0,0, $\frac{\sqrt{13}-1}{2}, \frac{-\sqrt{13}+1}{2}, \frac{\sqrt{13}+1}{2}, \frac{-\sqrt{13}-1}{2}$

Graph Energy = The addition of the absolute values of eigenvalues.

$GE_1 = 7.21$

➢ *Phase II*

- *Path of Phase II*

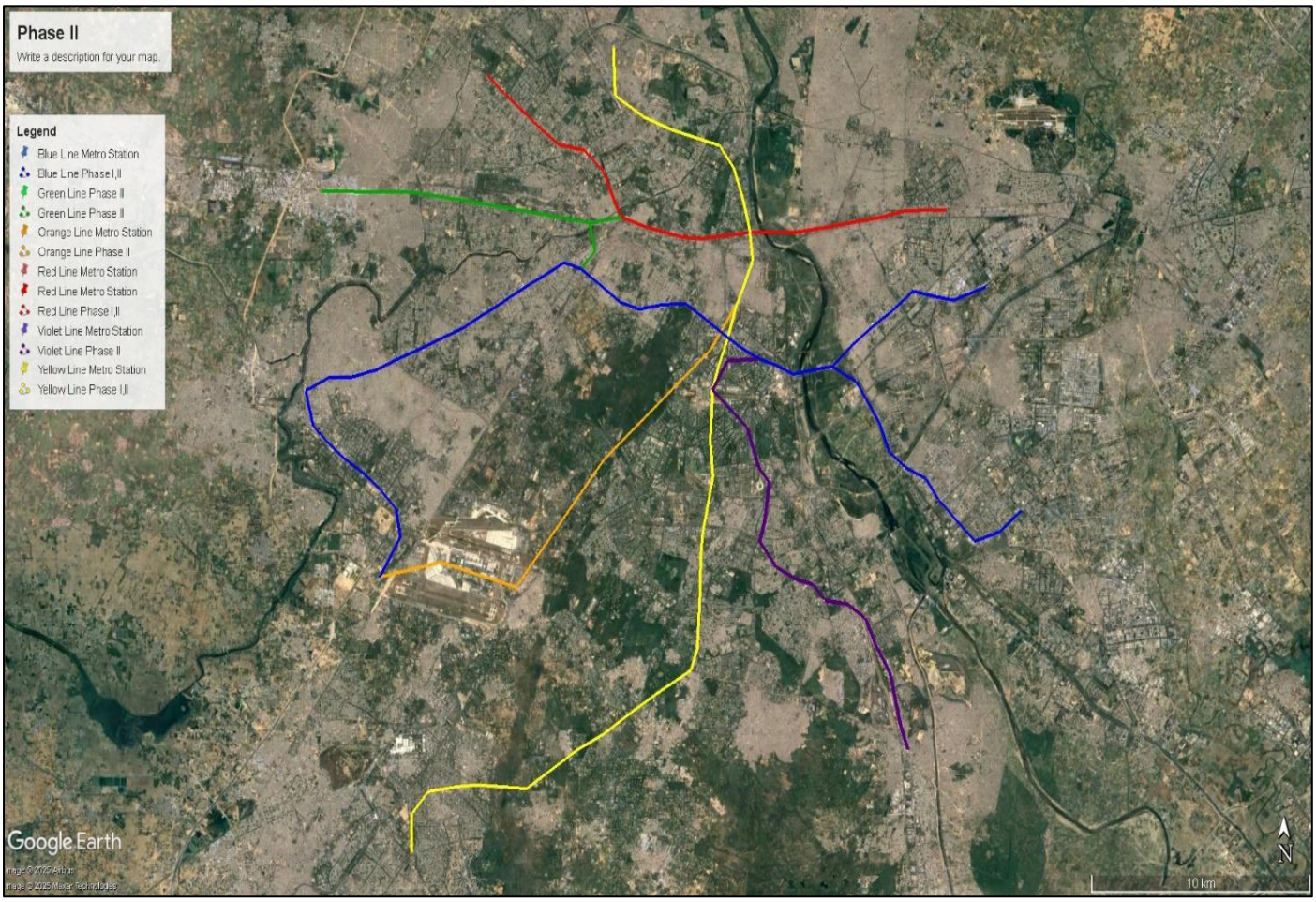

Fig 9 Path of Phase II
[Own Work]

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- *Graph of Phase II*

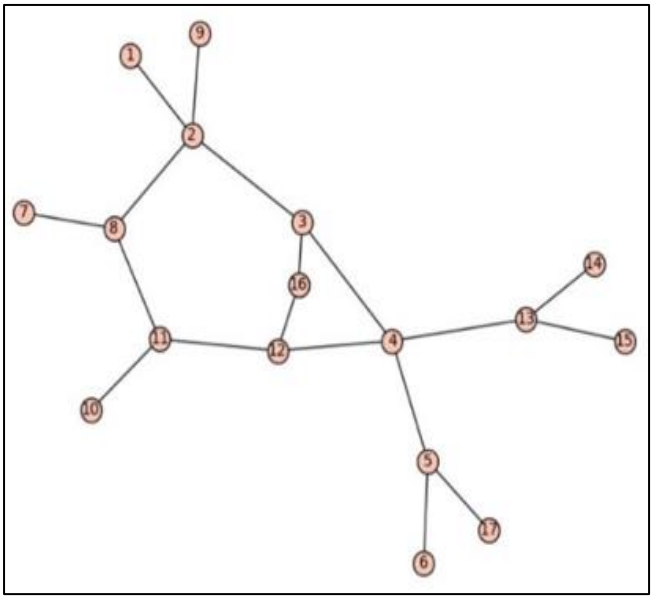

Fig 10 Graph of Phase II
[Own Work]

Only endpoints of metro lines and interchange stations are chosen for nodes/vertices.

For the phase II graph e= 18, v = 17, $d(1) = 1$, $d(2) = 4$, $d(3) = 3$, $d(4) = 4$, $d(5) = 3$, $d(6) = 1$, $d(7) = 1$, $d(8) = 3$, $d(9) = 1$, $d(10) = 1$, $d(11) = 3$, $d(12) = 3$, $d(13) = 3$, $d(14) = 1$, $d(15) = 1$, $d(16) = 2$, $d(17) = 1$

Table 2 Calculation for Phase II

Indices	Calculations	Result
$M_1(G)$ First Zagreb Indices	$d^2(1) + d^2(2) + d^2(3) + d^2(4) + d^2(5) + d^2(6) + d^2(7) + d^2(8) + d^2(9) + d^2(10) + d^2(11) + d^2(12) + d^2(13) + d^2(14) + d^2(15) + d^2(16) + d^2(17)$	98
$M_2(G)$ Second Zagreb Indices	$d(1).d(2) + d(2).d(3) + d(2).d(8) + d(2).d(9) + d(3).d(4) + d(3).d(16) + d(4).d(5) + d(4).d(12) + d(4).d(13) + d(5).d(6) + d(5).d(17) + d(7).d(8) + d(8).d(11) + d(10).d(11) + d(11).d(12) + d(12).d(16) + d(13).d(14) + d(13).d(15)$	128
α Alpha Index	$\alpha = \frac{e - v + 1}{2v - 5}$	0.068
β Beta Index	$\beta = \frac{e}{v}$	1.058
γ Gamma Index	$\gamma = \frac{e}{3v - 6}$	0.4

- *Adjacency Matrix:*

$$A = \begin{pmatrix}
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&1&0&0&0&0&1&1&0&0&0&0&0&0&0&0\\
0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&1&0&1&0&0&0&0&0&0&1&1&0&0&0&0\\
0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
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0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

Eigenvalues of adjacency matrix are 0,0,0,0,0, $\sqrt{2}$, $-\sqrt{2}$, -2.706,-2.035,-1.565,-1.329,-0.566,0.566, 2.706,2.035,1.565, 1.329.

Graph Energy = The addition of the absolute values of eigenvalues.

$GE_2 = 19.2304$

III. CONCLUSION

When we go from phase I to phase II, the values of the first Zagreb indices and the second Zagreb indices exponentially grow. But the value of the alpha and gamma indices makes a slight change from phase I to phase II.

Graph energy has a rapid change from phase I to phase II.

These results are useful for further research. Similar work has been done for other metro networks.

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REFERENCES

[1]. https://worldpopulationreview.com/cities/india/delhi
[2]. https://delhimetrorail.com/pages/en/introduction
[3]. https://delhimetrorail.com/pages/en/funding
[4]. Advani, M., & Tiwari, G. (2005). Evaluation of public transport systems: case study of Delhi Metro. *Transportation Research & Injury Prevention Programme*, *1*.
[5]. Biradar, Ambika (2020). Topological Indices of Semigraph. *International Journal of Engineering and Advanced Technology*, *9*(3), 1382-1384.
[6]. Gutman, I., Li, X., & Zhang, J. (2009). Graph energy. *Analysis of Complex Networks: From Biology to Linguistics*, 145-174.
[7]. Gutman, I., & Trinajstić, N. (2005). Graph theory and molecular orbitals. In *New concepts II* (pp. 49-93). Berlin, Heidelberg: Springer Berlin Heidelberg.
[8]. Somani, R. P., & Jethwani, V. (2023). Introducing new exponential Gourava indices for graphs. *International Journal of Mathematics and Statistics Invention*, *11*(3), 1-5.
[9]. Somani, R., Jethwani, V. (2023). Some new graph operations and their first and second Zagreb and Gourava indices. Journal of Emerging Technologies and Innovative Research 10(8), d567-d577.
[10]. Todeschini, R., & Consonni, V. (2009). *Molecular descriptors for chemoinformatics: volume I: alphabetical listing/volume II: appendices, references*. John Wiley & Sons.
[11]. West, D. B. (2001). *Introduction to graph theory* (Vol. 2, pp. 1-512). Upper Saddle River: Prentice hall.