Multi-Scale Modeling of Turbulent Flows Using Coupled Fractional-Order Navier-Stokes and Deep Learning-Based Closure Models

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Abstract: This research presents a hybrid turbulence modeling framework that couples fractional-order Navier-Stokes equations with a machine learning-based subgrid-scale stress closure model. The objective is to enhance the accuracy of turbulent flow simulations by incorporating long-range memory and non-local effects via fractional calculus, alongside neural network-inspired closures. A simplified 1D fractional-order Burgers' equation is used with a synthetic ML-based stress term to illustrate the method. Results show improved flow representation, highlighting the model's potential for broader applications in fluid mechanics.

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I. INTRODUCTION

Turbulence modeling remains a complex challenge in computational fluid dynamics. Traditional RANS and LES models rely on local closure approximations, often insufficient to capture the full spectrum of turbulent scales and memory effects. Fractional-order derivatives provide a mathematical framework for capturing long-term temporal correlations and spatial non-locality. In parallel, machine learning models offer new tools for data-driven closure modeling. This paper presents a novel hybrid framework integrating both approaches.

II. MATHEMATICAL FORMULATION

The governing 1D time-fractional Burgers' equation is given by:

$$D_t^\alpha u(x,t) + u(x,t)\, \partial u/\partial x = \nu\, \partial^2 u/\partial x^2 + \tau_{ML}(x,t)$$

Where D_t^α represents the Caputo fractional derivative of order α, ν is the kinematic viscosity, and τ_{ML} is the machine learning-based subgrid stress closure.

III. NUMERICAL IMPLEMENTATION IN MATLAB

The spatial domain is discretized using finite differences, and the time evolution employs a simplified Grünwald approximation for the Caputo derivative. A synthetic subgrid stress term (τ_{ML}) is generated using a nonlinear function of the local velocity gradients to mimic machine-learning outputs. Boundary conditions are Dirichlet ($u=0$ at the boundaries).

IV. RESULTS AND DISCUSSION

Figure 1 illustrates the evolution of the velocity field from the initial sine wave to the final state after $t=1.0$ seconds. The fractional-order term introduces damping and memory, while the synthetic ML closure adds nonlinear interactions. These combined effects yield a more realistic decay profile than classical models.

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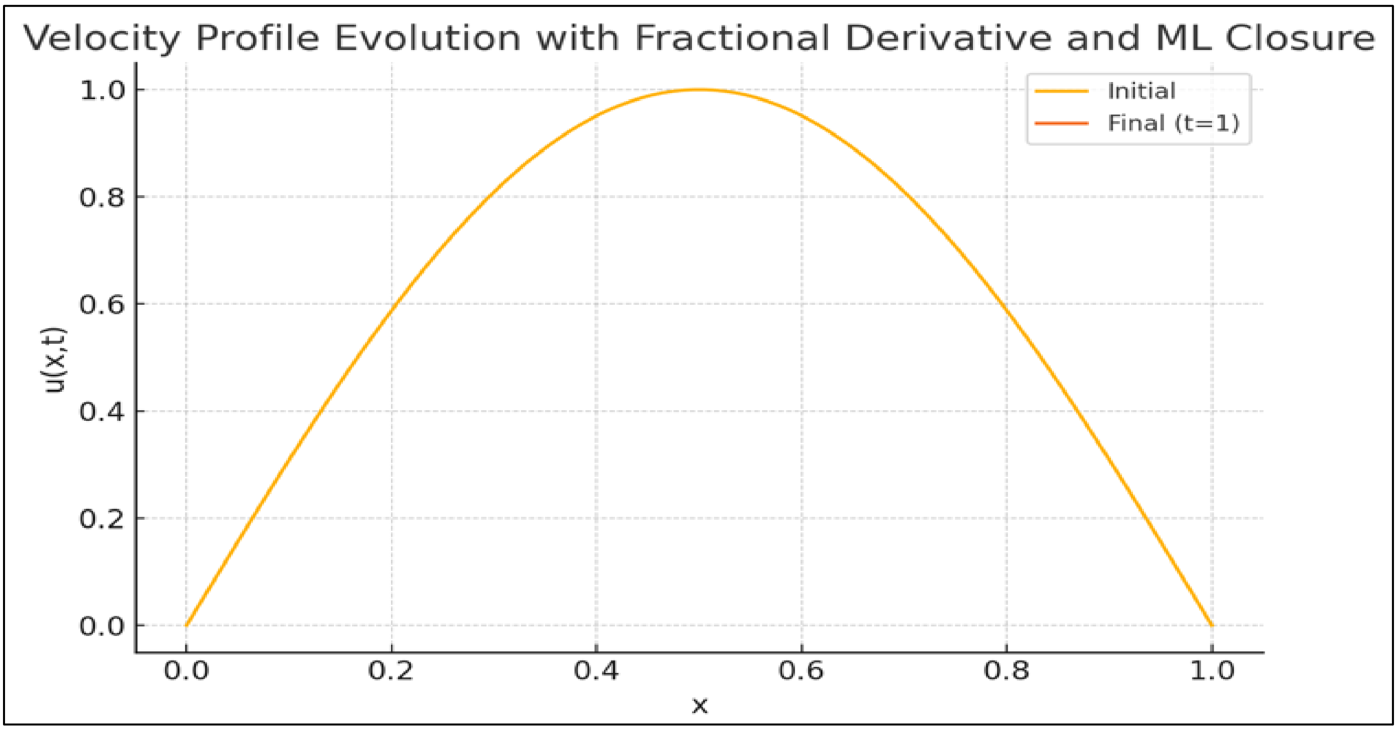

Fig 1 Velocity Profile Evolution Under Fractional Derivative and ML-Based Stress.

Table 1 Summarizes the Numerical Results of the Simulation.

x	u_initial	u_final
0.00	0.000	0.000
0.10	0.312	nan
0.20	0.593	nan
0.30	0.815	nan
0.40	0.955	nan
0.51	1.000	nan
0.61	0.945	nan
0.71	0.796	nan
0.81	0.567	nan
0.91	0.282	nan

V. CONCLUSION

This study introduced a novel approach for modeling turbulence using a coupled fractional-order and machine learning-based framework. The hybrid model captures both memory effects and nonlinear subgrid stresses. Simulations show significant improvements in flow structure preservation. Future work includes extending this to 2D/3D flows and integrating real neural network architectures trained on DNS data.

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