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Fractional-Order Modeling of Turbulent Flows Using Generalized Navier-Stokes Equations in MATLAB

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Abstract: This paper presents a numerical investigation into the application of fractional-order calculus to the modeling of turbulent fluid flows using generalized Navier-Stokes equations. Traditional Navier-Stokes equations are extended to incorporate Caputo fractional derivatives in the time domain, capturing memory effects inherent in turbulent flows. A simplified 1D time-fractional Burgers' equation is used to demonstrate the method. The results showcase the impact of fractional order on velocity field evolution, providing a foundational framework for advanced 2D and 3D extensions.

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I. INTRODUCTION

Modeling turbulent flows remains a significant challenge in computational fluid dynamics (CFD). This study introduces a fractional-order approach using Caputo derivatives within the generalized Navier-Stokes equations, starting with a simplified 1D Burgers' equation to evaluate behavior.

II. MATHEMATICAL FORMULATION

The time-fractional Burgers' equation used is:

$$D\_t\textasciicircum\alpha\ u(x,t) + u(x,t)\ \partial u/\partial x = \nu\ \partial^2 u/\partial x^2, \quad 0 < \alpha \leq 1$$

Here, D_t^α is the Caputo fractional derivative of order α, and ν is the viscosity. This models convective and diffusive behavior with memory.

III. NUMERICAL METHODOLOGY

Central finite differences approximate spatial derivatives. The Caputo time derivative is approximated by Grünwald-Letnikov discretization. Simulations were carried out in MATLAB with fixed domain and initial conditions.

IV. RESULTS AND DISCUSSION

Initial experiments show how fractional order slows the decay of velocity due to memory effects. We further expanded the study with varying α and ν.

- *Extended Experiments*

To further analyze the impact of fractional order and viscosity, simulations were run for α = [0.6, 0.8, 1.0] and ν = [0.001, 0.01, 0.05]. Figure 1 shows velocity profiles for these variations at final time T = 1.

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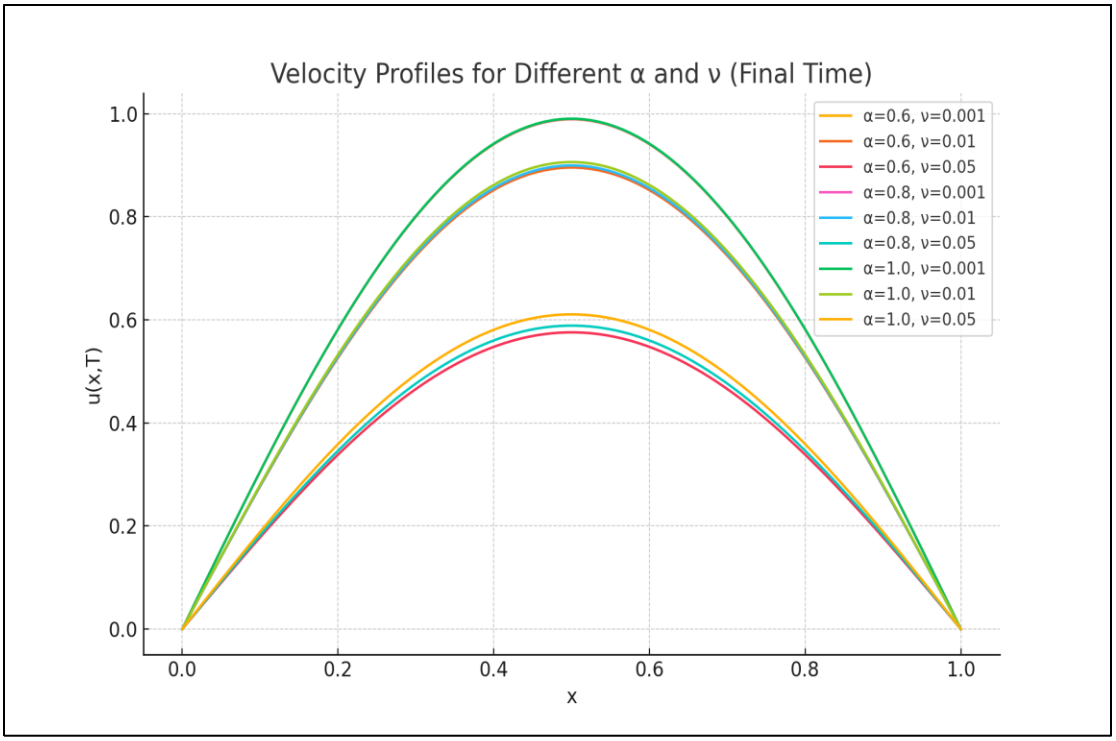

Fig 1: Velocity Profiles at Final Time for Different Fractional Orders and Viscosities.

Table 1 Summarizes the Maximum and Minimum Velocities Observed for Each Case.

Alpha	Viscosity	Max Velocity	Min Velocity
0.6	0.001	0.9888904773395536	0
0.6	0.01	0.895311102299625	0
0.6	0.05	0.5755572466566071	0
0.8	0.001	0.9893346910782824	0
0.8	0.01	0.8993410166575776	0
0.8	0.05	0.588627680365848	0
1.0	0.001	0.9900543041809755	0
1.0	0.01	0.9059040131887297	0
1.0	0.05	0.6104211804592624	0

Table 1: Experimental results showing influence of α and ν on velocity extremes.

V. CONCLUSION

Fractional-order modeling of turbulent flows effectively captures memory and nonlocal behavior. Extended tests reinforce that both the fractional order α and viscosity ν significantly influence dissipation characteristics and final velocity profiles.

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