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Mathematical Modeling and Control of Non-Rigid Axisymmetric Parts Processing

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Abstract: This article investigates the control of technological system parameters in the processing of elastically deformed non-rigid axisymmetric parts. The developed mathematical models describe the relationships between machining accuracy and key technological factors. Methods for improving accuracy by introducing positive feedback on cutting force, which eliminates static errors, as well as approaches to minimizing the system's sensitivity to changes in technological environment parameters, are presented. An analysis of the dynamic characteristics and stability of control systems using sensitivity theory was conducted. Methods for optimizing control structures to improve their speed and processing quality, including the application of integral quadratic criteria, are considered. The research results confirm the possibility of adapting the proposed approaches for various metalworking operations, ensuring a significant improvement in the accuracy and quality of parts processing

Keywords: *Technological System, Elastically Deformed Parts, Machining Accuracy, Processing Parameters, Cutting Force, Dynamic Characteristics, Control Structure Optimization, Mathematical Modeling.*

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I. INTRODUCTION

Modern requirements for the quality and precision of machining parts in mechanical engineering necessitate continuous improvement of automatic control systems for technological processes. One of the key factors determining the final machining accuracy is the elastic displacements and elastically deformed state of the technological system (TS) elements, which significantly influence the relative position of the tool and the workpiece.

This work examines models and control methods that allow for increased processing accuracy and quality by accounting for the elastic properties of the TS elements and introducing additional connections into the control system. In particular, approaches to the formation of optimal control system structures, considering the influence of both control and disturbance effects, are investigated. It is shown that traditional control systems built on static principles are susceptible to errors caused by changes in workpiece parameters and cutting conditions.

Special attention is paid to the introduction of additional positive feedback on the cutting force, which allows for the compensation of static errors and significantly increases the accuracy of tool positioning. Methods for designing systems with reduced sensitivity to parameter variations based on sensitivity theory are also considered, which is especially relevant for high-precision machining conditions.

Analysis of the dynamic characteristics of such systems using numerical modeling confirms the effectiveness of the proposed solutions: rapid attenuation of forced oscillations and stabilization of elastic displacements are achieved. An integral quadratic criterion is used for control, which ensures high quality of transient processes and processing accuracy.

The proposed methods and control systems can be effectively applied not only in turning operations, but also in other cutting processes, which demonstrates their versatility and practical significance.

II. LITERATURE REVIEW

The studies by S.V. Gruby [1] and A.N. Rykunov [5] are dedicated to examining the features of surface layer formation, taking into account tool wear, where conditions for stable cutting and minimum parameters of material removal are considered. An important direction is also the numerical modeling of complex surface cutting, which is described in detail in the publication by Yu.E. Petukhov [2]. The combined and automatically regulated processes discussed in the work of V.N. Podurayev [3] contribute to more flexible and adaptive control of machining parameters.

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A significant factor determining the accuracy and stability of machining is the dynamics of the mechanical system. The fundamentals of mechanical vibration theory are presented in the work of Panovko Ya.G. [4], while the applied aspects of grinding process vibrations are investigated in the works of Novikov F.V. and Berezhny R.A. [13], as well as Lizogub V.A. [16], where the influence of spindle assembly parameters on accuracy is emphasized.

The development and application of automatic machine tool systems are reflected in the fundamental monograph by Push V.E. and co-authors [6], while the issues of identification and intelligent control of complex technical objects are addressed in the works of Rastrigin L.A., Okhtilev M.Yu., and other researchers [8, 14].

Modern modeling technologies, including simulation and structural models of cutting and grinding processes, are actively developing, as confirmed by the research of Voronov S.A. and Medvedev A.S. [10, 11]. Additionally, several works focus on the application of CAD and computer calculation systems such as SolidWorks [12, 17], which ensures the integration of process design and analysis in a unified digital environment.

Mathematical methods of analysis and modeling, which enable a deeper understanding of the dynamics of technical systems, have gained particular importance, as demonstrated in the work of R. Sadullayev and N. Ravshanov [7]. A comprehensive overview of technological processes and the design of metal-cutting machines is provided in the works edited by V.S. Korsakov and D.I. Reshetov [9, 15], forming a solid theoretical foundation for modern research.

Potential for improving machining accuracy and quality can be identified when constructing optimal control system structures, as the systems for automatic control of elastic displacements in technological systems [10, 11, 12] are static in terms of both control and disturbance effects. Changes in these effects lead to errors in the relative positioning of the workpiece and the cutting tool. Complete or partial elimination of these errors enhances the accuracy of the control system operation and, consequently, the machining of parts. This is achieved by introducing an additional positive feedback loop based on cutting force into the control system for elastic displacements of the technological system.

In works [13, 14], a method for fine-tuning technological systems through control has been proposed and theoretically substantiated. This method involves introducing additional positive feedback on the cutting force into the control system, which allows for the elimination of static errors in both control and disturbance effects. This is achieved by adjusting the magnitude of the longitudinal feed and the parameters of the elastically deformed state of non-rigid parts in the technological system. Conditions were obtained for determining the structure and parameters of additional feedback based on the cutting force, ensuring the elimination of static errors in the control system.

Designing control systems that maintain operability under uncontrolled variations of technological system parameters using sensitivity theory allows the control system to acquire certain properties of insensitivity to parameter changes.

In the study [15], an analysis of the dynamic operating conditions of control systems for elastic displacements in technological systems was conducted using sensitivity theory methods [16]. As a result, frequency ranges at which stability loss occurs were identified, indicating the need to reduce the control system's sensitivity to changes in workpiece allowance. To eliminate the influence of workpiece allowance on the steady-state error and to ensure stability, a corrective element is introduced into the elastic displacement control system in the form of negative feedback on the cutting force - a transfer function determined by the system structure. The possibility of constructing control systems with low sensitivity to parameter changes during turning operations has been substantiated. The developed methods and control systems can be applied to other metal-cutting operations.

Improved speed and quality of control systems are achieved by using integral quadratic estimation and enhanced integral quadratic estimation as accuracy criteria in managing elastic displacements and elastically deformed states [17]. The considered accuracy criterion was implemented through an automatic control system for a lathe, and analysis of the transient processes obtained through numerical modeling in the control system indicates a sufficiently rapid attenuation of forced oscillations of elastic movements in the technological system and high processing accuracy of parts.

Thus, the conducted analysis of sources indicates a multifaceted approach to the study of cutting and grinding processes. The relevance of further research lies in the need to create comprehensive models that take into account both mechanical and physical parameters as well as intelligent control methods to improve the quality and efficiency of processing.

III. METHODOLOGY

When controlling technological systems for processing non-rigid axisymmetric parts based on changes in their elastically deformed state, regulatory influences are used as individual or combinations of controlled force effects according to the adopted classification: central and eccentric tension, eccentric compression; control of additional force effects aimed at compensating for force factors from the cutting process; bending moments on supports; and control of bending-torsional force deformation [1, 2, 3, 4, 5].

The mathematical model of the technological system in steady-state modes is formed as a functional dependency, reflecting the influence of regulatory and disturbing effects on the magnitude of elastic deformations of the part in the considered cross-section. Based on the laws of solid deformable body mechanics [6, 7], functional dependencies for technological systems with various types of elastically deformed states of non-rigid parts were obtained.

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The mathematical models of the technological system (TS) with controlled elastic-deformed state in steady-state regimes, presented in the form of deflection functions, were obtained under the assumption that the bending force acting on the workpiece is an external variable independent of the TS's elastic deformations.

This approach is based on disregarding the closed nature of the elastic system through the cutting process and does not introduce significant errors in the results of analyzing the statistical characteristics of the control object. At the same time, analysis shows that it is impossible to construct a viable mathematical model of the control object in transient modes without considering the peculiarities of the processes in the machining zone and the closed-loop nature of the TS through the cutting process.

Additional force influences formed on the TS to control the elastic-deformed state of the workpiece generally lead to the appearance of additional deformations of the elastic system for each of the coordinates, which is accounted for by introducing the second term into the last relation.

$$\Delta Y_{\xi}(S) = K_{\xi}(S)\cdot \Delta P_{\xi}(S) + \Delta Y_{f_{\xi}}(S) \tag{1}$$

The mathematical model of the object under consideration - the technological system (TS) with control of the elastic-deformed state of a non-rigid part - is formed based on the principles for constructing dynamic models [133] of machining systems. In this case, the characteristic features of the non-rigid parts machining process are taken into account by introducing the corresponding coupling equations [8, 9, 2, 4], reflecting the relationship between the additional elastic deformations $\Delta y_{f\xi}$ in equation (1) and the force-controlling influences.

It is advisable to represent the equivalent (total) elastic deformations of the TS during the processing of non-rigid parts in the form of two components.

$$y_{\xi} = y_{\xi c} + y_{\xi \partial} \tag{2}$$

Where $y_{\xi c}$ and $y_{\xi д}$ - corresponding elastic deformations of the machine - fixture - tool and workpiece for each coordinate; $\xi \in \{x,y,z\}$. The first term in this expression for the considered TS is, as a rule, an order of magnitude smaller than the second and can be neglected.

Elastic deformations of the TS in the radial direction y_y in accordance with the deflection equations in steady-state modes, without considering the closedness of the control object, can be considered as a deterministic nonlinear function of the workpiece parameters *L*, *d*, *EI*; components of the cutting forces P_z, P_y, P_x; coordinate *X* of the cutting force application along the length of the workpiece and various regulating influences in the form of a tensile force P_{x1} ; an eccentric tensile force creating two regulating influences P_{x1} and $M = P_{x1_0}\cdot e$; one or several additional counteracting forces $P_{доп.i}$; bending moments M_i; twisting moment $M_{кр}$, their combinations, and others:

$$Y_y = f\left(L, d, EI, P_z, P_y, P_x, P_{x1}, e, P_{доп.i}, M_i, M_{кр}, X\right). \tag{3}$$

Considering that at realistic values of longitudinal feed rate, the rate of change of the *X* coordinate is relatively small, when analyzing transient processes "in small" terms, the change of the *X* coordinate as a function of time can be neglected. Then equation (2.35) for increments in operator form takes the following form:

$$Y_y(s) = K_{xy}\cdot P_x(s) + K_{yy}\cdot P_y(s) + K_{zy}\cdot P_z(s) +$$
$$K_{Px1}\cdot P_{x1}(s) + K_e\cdot e(s) + K_{P_{доп.i}}$$
$$\cdot P_{доп.i}(s) + K_{Mi}\cdot M_i(s) + K_{M_{кр}}\cdot M_{кр}(s), \tag{4}$$

Where the double indices with the coefficients *K* indicate that the coefficients K_{xy}, K_{zy} show the influence of the increment of components P_x, P_z on the elastic deformation increment along the coordinate *Y;* $K_e = K'_e \cdot P_{x1_0}$. The transfer coefficients of linearized equations are defined as partial derivatives of the deflection functions with respect to the corresponding variable. Thus, for example, for a technological system of processing under the action of a tensile force P_{x1}, creating an elastically deformed state, the transfer coefficients are obtained in the form:

$$K_{yy} = \left(\frac{dy_y}{dP_y}\right)_0 = \frac{L^3[1-\cos(2\pi x_0/L)]^2}{2\pi^2(4\pi^2 EI + P_{x1_0}\cdot L^2)} \tag{5}$$

$$K_{P_{x1}} = \left(\frac{dy_y}{dP_{x1}}\right)_0 = -\frac{P_y\cdot L^5[1-\cos(2\pi x_0/L)]^2}{2\pi^2(4\pi^2 EI + P_{x10}\cdot L^2)} = -\frac{y_{y0}\cdot L^2}{4\pi^2 EI + P_{x1_0}\cdot L^2} \tag{6}$$

Where P_{x1_0} and Y_{y_0} - are the values of the tensile force and elastic deformation of the part along the *Y* axis at the linearization point (values of variables relative to which the increments of the variables are given).

In the particular case under consideration, the remaining transmission coefficients included in equation (4) are equal to 0. The transfer coefficients corresponding to other technological systems for various loading and securing methods during the processing of elastically deformed parts, obtained in a similar way, are presented in Table 1 - column 2; X0 is the coordinate of the cutting tool position along the machining length at the linearization point.

Elastic deformations of the technological system along the Z-coordinate influence the change in cutting depth. When machining non-rigid parts of small diameter, in some cases, this effect can be quite significant. The increase in cutting depth bz (equivalent to additional elastic deformation along the Y axis), caused by elastic deformations yz along the Z axis (1):

$$b_z = R - R_{\cos(\alpha)}, \tag{7}$$

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Moreover, considering the small angle α

$$\arcsin(y_z/R) \approx y_z/R$$

After linearization, the expression (2.39) for increments is written as (the increment sign here, as above, is omitted)

$$b_z(s) = K_{bz} \cdot y_z(s), \tag{8}$$

Where $K_{bz} = \sin(y_{z0}/R) \approx y_{z0}/R$ is the transfer coefficient that establishes the relationship between the cutting depth increment b and the cutting force component P_z.

Additional elastic deformations yfx , yfz along the X and Z coordinates due to the considered force control actions typically have an insignificant impact on the dynamic properties of the controlled object and can be neglected.

The obtained properties of the controlled object model fundamentally allow for achieving higher processing quality indicators when managing the elastically deformed state of the workpiece.

IV. CONCLUSION

In the course of this research, control methods for technological systems processing elastically deformed non-rigid axisymmetric parts were developed and theoretically substantiated. The proposed approaches, including the use of additional positive feedback on the cutting force, allow for the elimination of static errors and increased machining accuracy. The introduction of a corrective element in the form of negative feedback ensures the stability of control systems and reduces sensitivity to changes in workpiece parameters.

The application of sensitivity theory has made it possible to identify critical frequency ranges leading to loss of stability and propose methods for their minimization. Using integral quadratic criteria as a control quality indicator demonstrated the effectiveness of the proposed systems in turning operations and other types of cutting. The obtained results create a basis for the further development of automated control systems with improved accuracy, speed, and stability characteristics.

Table 1 Models and Coefficients of Linearized Equations for some cases of part Stretching

Model No. **Regulatory impact Source of information**	**Transfer coefficient of linearized equation**
1	**2**
Model 1 Central stretching	$K_{yy} = (\frac{dy_{y\partial}}{dP_y})_0 = [(\beta\alpha LchdL - sh\beta\alpha L)(sh\alpha x_0 - \alpha x_0)/\alpha Px_1(\alpha Lch\alpha L -$ $- sh\alpha L)] - [L(\beta sh\alpha L - sh\beta dL)(ch\alpha x_0 - 1)/P_{x1}(\alpha Lch\alpha L - sh\alpha L)];$ $K_{Px1} = (\frac{dy_{y\partial}}{dP_{x1}})_0 = \{P_y[\beta L\alpha' ch\alpha L + \beta L^2\alpha' sh\alpha L - \beta L\alpha' ch\beta\alpha L]\cdot$ $\cdot(sh\alpha x_0 - \alpha x_0) + P_y(x_0\alpha' ch\alpha x_0 - x_0\alpha')[\beta\alpha Lch\alpha L - sh\beta\alpha L]\} \div$ $\div \alpha P_{x1}\cdot(\alpha Lch\alpha L - sh\alpha L) - \{\alpha^2 LchL - \alpha sh\alpha L + \alpha P_{x1}(\alpha' Lch\alpha L +$ $+ \alpha L^2\alpha' sh\alpha L - \alpha' Lch\alpha L)\}\times P_y[\beta\alpha Lch\alpha L - sh\beta\alpha L](sh\alpha x_0 - \alpha x_0)\div$ $\div \alpha^2 P^2{}_{x1}(\alpha Lch\alpha L - sh\alpha L)^2 - \{P_y L[(\alpha' L\beta ch\alpha L - \alpha'\beta Lch\alpha\beta L)\cdot$ $\cdot(ch\alpha x_0 - 1) + (\beta sh\alpha L - sh\alpha\beta L)](\alpha' x_0 sh\alpha x_0 - 1)\times$ $\times 1/Px_1(\alpha Lch\alpha L - sh\alpha L) - \{\alpha Lch\alpha L - sh\alpha L + \alpha' P_{x_1}\alpha L^2 sh\alpha L +$ $+ \alpha' Lch\alpha L\}P_y L\cdot(\beta sh\alpha L - sh\alpha\beta L)(ch\alpha x_0 - 1)/P^2{}_{x1}(\alpha Lch\alpha L - sh\alpha L)^2;$ $\beta = \frac{L-a}{L}; \alpha = \sqrt{\frac{P_{x1}}{EI}}; \quad \alpha' = \frac{1}{2\sqrt{P_{x1}EI}}.$
Model 2 Central stretching (collet chuck)	$K_{yy} = (\frac{dy_{y\partial}}{dP_y})_0 = \frac{L^3[1-\cos(2\pi x_0/L)]^2}{\pi^2[8\pi^2 EI + 2P_{x1}L^2]};$ $K_{Px1} = (\frac{dy_{y\partial}}{dP_{x1}})_0 = \frac{P_y L^5[1-\cos(2\pi x_0/L)]^2}{2\pi^2[4\pi^2 EI + P_{x1}L^2]^2};$

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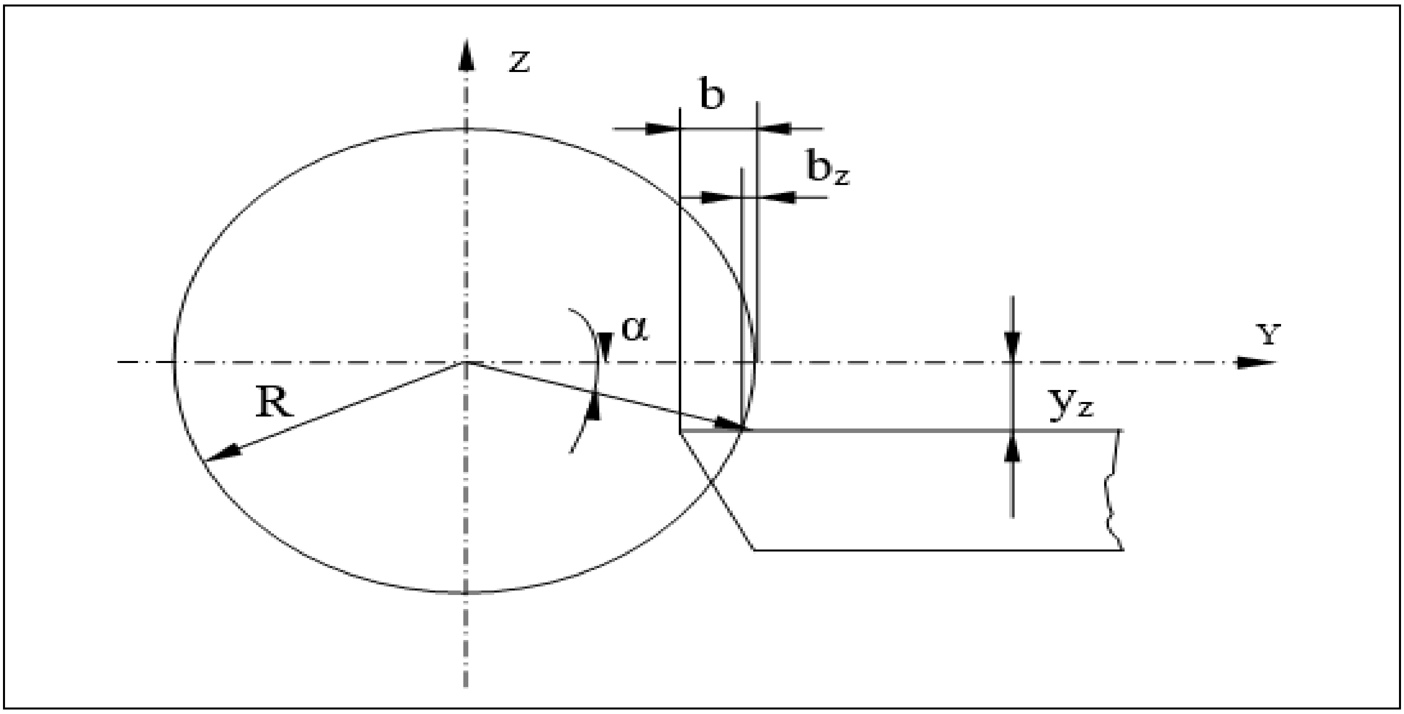

Fig 1 Diagram of the Increase in Cutting Depth caused by Elastic Deformations along the Z Axis.

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