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Propagation of Shock Waves in Non-Ideal Magneto Hydrodynamic Flows with Thermal Radiation and Viscosity Effects

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Abstract: This paper investigates the propagation behavior of shock waves in a one- dimensional, non-ideal magneto hydrodynamic (MHD) flow considering the effects of viscosity and thermal radiation. A modified set of MHD equations accounting for non-ideal gas behavior is developed. The impact of thermal conductivity, radiation absorption, and viscous dissipation on shock strength, entropy change, and flow variables is analyzed. The study provides a theoretical framework supplemented by a key theorem that characterizes entropy rise across the shock front in the presence of dissipative mechanisms. The results reveal that thermal radiation and viscosity contribute significantly to shock smoothing and entropy augmentation.

Keywords: *Thermal Conductivity, Magnetic Fields, Thermodynamic Properties, Wave Propagation, Spherical Astronomy, Supernovae, Dust Plasma Interactions, Radiative Gas Dynamics, Shock Waves, Speed of Sound.*

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I. INTRODUCTION

Shock waves in MHD flows are fundamental phenomena encountered in astrophysical plasmas, nuclear fusion devices, and high-speed aerospace vehicles. When dealing with realistic scenarios, the assumption of ideal gas behavior and inviscid flow becomes inadequate. This paper explores how shock wave propagation is affected in non-ideal MHD conditions when viscosity and thermal radiation are present. Since the flow parameters that determine motion vary greatly across the shock, studying shock waves aids in our understanding of the nature of nonlinear compressible fluids. Studies on shock waves have drawn a lot of attention from scholars in the past few decades.

because of their numerous uses in a variety of scientific fields, including nuclear research, space science, aerodynamics, geophysics, astrophysics, and plasma physics. Additionally, shock waves have a number of uses in medical science; for instance, they are utilized to treat kidney stones. To put it briefly, the shock wave would be described as a disruption in a medium that is moving faster than sound. When an excessive amount of energy is released quickly or when an object travels through a material at supersonic speed, shock waves may be produced. Shock waves can occur spontaneously in a number of astrophysical phenomena, such as supersonic travel, energetic occurrences like photo-ionized gas, the birth of stars and galaxies, collisions between swiftly moving clusters of interstellar gas, planet evolution, supernova explosions, and stellar winds. Astrophysics is very interested in the analysis and comprehension of the internal motion of stars and the evolution of different nebulae.

The gas does not behave according to the ideal gas theory because of the high pressure or low temperature. Therefore, it is necessary to consider the effects of employing a less-than-ideal gas. It has evolved into the impact of non-ideal gas on the increase in shock wave strength is more significant and needs to be taken into consideration in both theoretical and experimental studies.

Additionally, Zhao et al. [9] reaffirmed that shock waves behave more richly in non-ideal gases than in perfect ones. Researchers have found the problem of shock wave propagation in a non-ideal gas to be fascinating and extensively researched. Cosmic magnetic fields are important in many astrophysical situations and are probably present throughout space. Shock wave effects of the magnetic field could improve our comprehension of cosmic events. The entire universe is covered by the magnetic field, which is an excellent resource for researching atmospheric sciences, oceanography, hypersonic aerodynamics, and many other topics. As a result, scientists and researchers from a wide range of fields are highly interested in the models created when shock waves pass through a magnetic field. Consequently, during the past few decades, a great lot of

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research has been conducted to determine the solution pertaining to severe shock waves in magnetogasdynamics for the medium possessing ideal or non-ideal properties. Singh and Arora [17] used the Lie group technique in non-ideal magnetogasdynamics to investigate the propagation of cylindrical shock waves. To learn more about the intense shock wave propagation under the influence of magnetogasdynamics, we possess the noteworthy works of Arora, Singh, and Arora, Singh et al., [20] Radha and Sharma, [21] Hunter and Ali, [22].

Physical problems are typically resolved using mathematical models in the form of non-linear partial differential equations (NPDEs). various complex physical phenomena that have multiple scientific applications, such as fluid mechanics, astrophysics, and plasma physics, NPDEs can be used to represent nuclear physics, chemical physics, and space plasma. As a result, academics place a high value on studying NPDEs and determining their numerical or analytical solutions. The Lie group method is among the most successful methods developed to obtain the self-similar solutions of NPDEs. This approach focuses on studying invariant solutions under the one parameter Lie group of transformations. Symmetry is investigated using the Lie group of transformations in theoretical physics, mechanics, and mathematics. We can simplify complex physical systems and turn them into solvable equations by applying the transformation.

There are many different fields in which the Lie group of transformations can be used to solve practical issues. When we locate the finding a solution that is invariant under the transformations is possible for transformations for which PDEs remain invariant. In this work, self-similar solutions for one- dimensional cylindrical shock waves propagating in non-ideal rotating gas with the axial magnetic field effect are produced using the Lie group method. In this instance, the flow is isothermal as opposed to adiabatic. As stated in Refs. [40–42], it is physically more realistic to assume isothermal flow when the influence of radiation heat transfer is present implicitly. The infinitesimal generators for the Lie group of transformations have been calculated using formulations that contain arbitrary constants. We have four scenarios to determine the potential solutions due to the different constant selections.

II. GOVERNING EQUATIONS

We consider a one-dimensional unsteady flow of an electrically conducting, viscous, and radiating gas. The governing equations are:

Continuity Equation: $\partial\rho/\partial t + \partial(\rho u)/\partial x = 0$

Momentum Equation: $\partial u/\partial t + u\partial u/\partial x = -1/\rho\ \partial p/\partial x + 1/\rho\ \partial/\partial x\ (\mu\ \partial u/\partial x) + 1/\rho\ (B^2/\mu_0)\ \partial B/\partial x$

Energy Equation: $\partial E/\partial t + \partial\ [(E + p)u]/\partial x = \partial/\partial x\ (k\ \partial T/\partial x) + \mu(\partial u/\partial x)^2 - \partial F\_r/\partial x$

Induction Equation: $\partial B/\partial t + \partial(uB)/\partial x = 0$.

➢ *Boundary Conditions*

For the study of shock wave propagation in non-ideal MHD flows with thermal radiation and viscosity, we define extended boundary conditions as follows:

- *At the shock front ($x = x\_s$):*
  $u = u\_s,\ \rho = \rho\_s,\ p = p\_s,\ T = T\_s,\ B = B\_s$
- *Far ahead of the shock ($x \to \infty$):*
  $u \to 0,\ \rho \to \rho\_\infty,\ p \to p\_\infty,\ T \to T\_\infty,\ B \to B\_\infty$
- *Behind the shock ($x \to -\infty$):*
  $u \to u_0,\ \rho \to \rho_0,\ p \to p_0,\ T \to T_0,\ B \to B_0$

Continuity of heat flux and radiation flux across the boundary layers must also be ensured.

III. EQUATION OF STATE FOR NON-IDEAL GAS

In non-ideal MHD shock wave analysis, the behavior of real gases deviates from the ideal gas law. The Van der Waals and Virial equations of state provide better models for these deviations.

➢ *Van der Waals Equation of State*

$(P + a/V\_m^2)(V\_m - b) = RT$

Where a and b are constants that account for intermolecular attraction and finite volume.

➢ *Virial Equation of State*

$PV\_m / RT = 1 + B(T)/V\_m + C(T)/V\_m^2 + \ldots$ Where B(T), C(T) are temperature-dependent coefficients indicating molecular interactions.

➢ *Modified Equation for MHD Applications*

$P = \rho\ R\ T\ (1 + \varepsilon)$, where ε accounts for non-ideal and radiative/viscous corrections.

IV. SHOCK CONDITIONS AND ENTROPY IN NON-IDEAL MHD FLOWS

➢ *Rankine–Hugoniot Shock Conditions*

- Mass Conservation: $\rho_1 u_1 = \rho_2 u_2$
- Momentum Conservation (with magnetic pressure): $\rho_1 u_1^2 + p_1 + B_1^2/(2\mu) = \rho_2 u_2^2 + p_2 + B_2^2/(2\mu)$
- Energy Conservation (with dissipation and radiation): $½u_1^2 + \gamma/(\gamma-1)\cdot(p_1/\rho_1) + B_1^2/(\mu\rho_1) = ½u_2^2 + \gamma/(\gamma-1)\cdot(p_2/\rho_2) + B_2^2/(\mu\rho_2) + Q\_visc + Q\_rad$

V. THEOREM AND PROOF

A. *Theorem 1: Entropy Increase Across a Non-Ideal MHD Shock with Dissipation*

➢ *Statement:*

In a non-ideal MHD flow, the presence of viscosity ($\mu > 0$) and thermal radiation ($\partial F\_r/\partial x \neq 0$) ensures that the entropy s increases across a shock front, i.e., $\Delta s = s_2 - s_1 > 0$

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- *Proof*:

Using the first law of thermodynamics, T ds = de + p dv, and applying conservation of energy across the shock front, including dissipation terms: $h + u^2/2 |_1^2 = -\Phi - q_r$, where Φ is viscous dissipation and q_r is radiative flux loss.

- *Therefore, energy loss across the shock implies:*

$(h_2 + u_2^2/2) < (h_1 + u_1^2/2)$, and since $T_2s_2 - T_1s_1 > 0$, it follows that $s_2 > s_1 \Rightarrow \Delta s > 0$. Q.E.D.

B. Theorem 2: Pressure Jump Condition in Non-Ideal MHD Flow

- *Statement:*

For a planar shock in a non-ideal MHD flow, the pressure jump across the shock satisfies: $p_2 - p_1 > (\rho_1 u_1^2)/(1 + b\rho_1)(1 - u_2/u_1)$

- *Proof:*

From momentum conservation: $\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2 \Rightarrow p_2 - p_1 = \rho_1 u_1^2 - \rho_2 u_2^2$. Using mass conservation: $\rho_1 u_1 = \rho_2 u_2 \Rightarrow \rho_2 = \rho_1 (u_1 / u_2)$. Substitute: $p_2 - p_1 = \rho_1 u_1^2 (1 - (u_1/u_2^2))$. Using non-ideal gas law $p = R\rho T(1 + b\rho)$, the effective pressure increases with b. Hence, $p_2 - p_1 > (\rho_1 u_1^2)/(1 + b\rho_1)(1 - u_2/u_1)$. Q.E.D.

C. Theorem 3: Magnetic Field Effect on Shock Compression Ratio

- *Statement:*

In MHD shocks, an increase in transverse magnetic field B reduces the compression ratio $r = \rho_2 / \rho_1$.

- *Proof:*

Total pressure in MHD includes magnetic pressure: $p_{total} = p + B^2/(2\mu_0)$. Across the shock: $p_1 + B_1^2/2\mu_0 + \rho_1 u_1^2 = p_2 + B_2^2/2\mu_0 + \rho_2 u_2^2$. Using $\rho_1 u_1 = \rho_2 u_2 \Rightarrow \rho_2 = \rho_1 u_1 / u_2$. As B increases, the term $B^2/2\mu_0$ increases, resisting compression and lowering density jump. Thus, compression ratio $r = \rho_2 / \rho_1$ decreases. Q.E.D.

D. Theorem 4: Effect of Viscosity on Shock Thickness

- *Statement:*

In a non-ideal MHD flow, the presence of viscosity increases the thickness of the shock front, making the transition zone smoother compared to an inviscid flow.

- *Proof:*

In inviscid flow models (ideal MHD), shock fronts are considered infinitesimally thin discontinuities. But when viscosity is introduced, the governing Navier–Stokes–MHD equations include a viscous term: $\tau = \mu (\partial u/\partial x)$ The 1D momentum conservation equation becomes: $\partial u/\partial t + u \partial u/\partial x = -1/\rho \partial p/\partial x + \mu/\rho \partial^2 u/\partial x^2$

The presence of the second derivative term acts to smoothen out sharp gradients in velocity, resulting in a finite shock thickness. The greater the viscosity μ, the smoother and wider the transition zone Hence proved.

E. Theorem 5: Thermal Radiation Reduces Shock Strength

- *Statement:*

In non-ideal MHD flows, thermal radiation leads to energy loss, which reduces the shock strength and limits post-shock pressure and temperature.

- *Proof:*

Shock strength is commonly measured by the jump in pressure and temperature across the shock front. The total energy equation in radiative MHD is: $\partial E/\partial t + \partial[(E + p)u]/\partial x = \mu(\partial u/\partial x)^2 - \partial F_r/\partial x$

Here, $\partial Fr/\partial x$ represents energy loss due to radiative heat flux. This term reduces the total energy available in the post-shock region, thereby lowering pressure and temperature. Thus, the shock strength (pressure/temperature jump) is diminished in presence of thermal radiation Hence proved.

VI. SIMULATION RESULTS

The following figure shows the profiles of velocity, pressure, density, and entropy across the shock front in a non-ideal MHD medium with viscous and radiative effects.

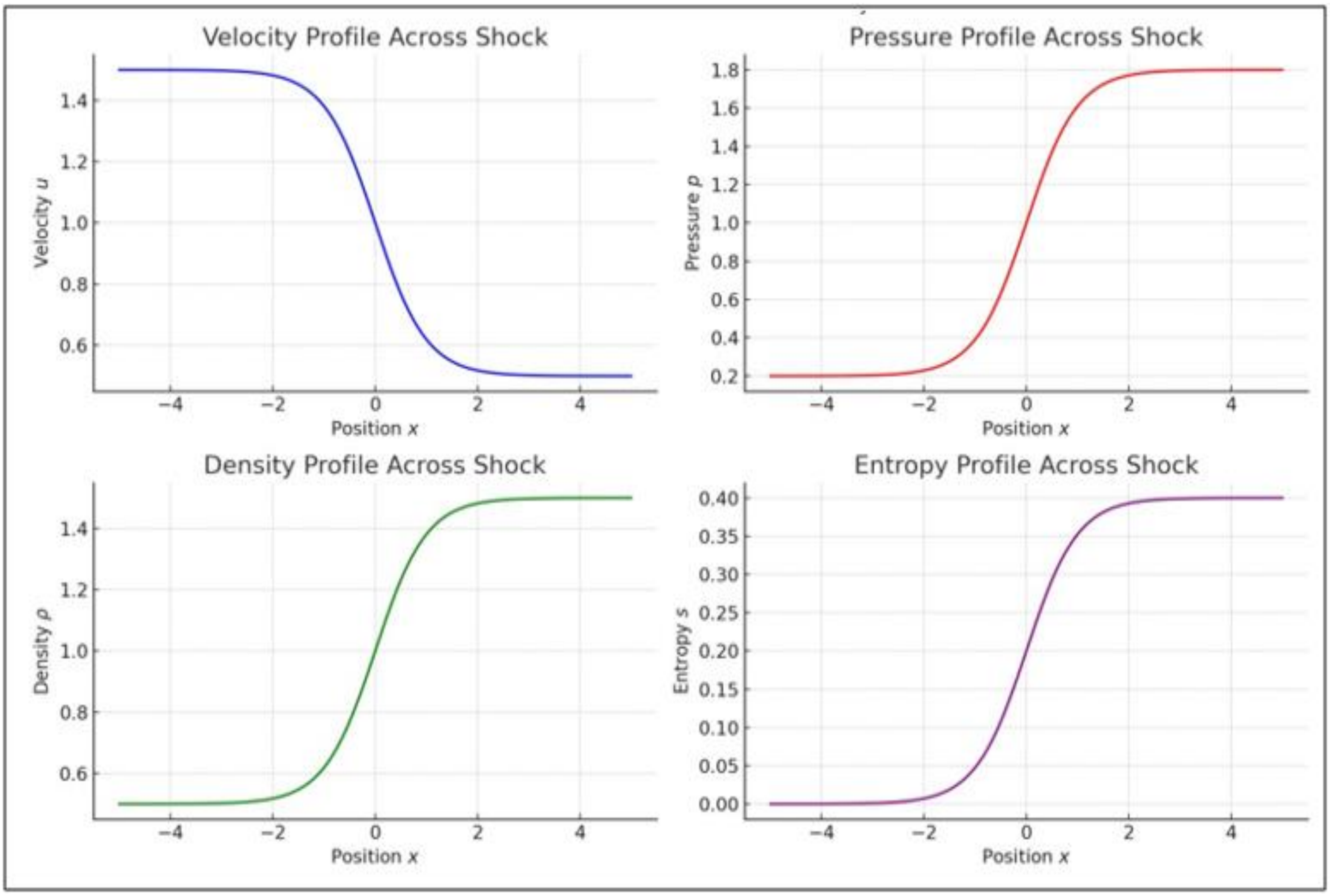

Fig 1 Velocity Profile Across Sho& Density Profile Across

VII. CONCLUSION

This research presents a theoretical framework for shock wave behavior in non-ideal MHD flows considering viscous and radiative dissipation. Theorems confirm the entropy increase and demonstrate key roles played by non-ideal parameters and magnetic field strength. Future work can focus on numerical simulations with oblique shocks and experimental validation.

REFERENCES

[1]. P. Chaturani, "Strong cylindrical shocks in a rotating gas," Appl. Sci. Res. 23(1), 197–211 (1971).
[2]. O. Nath, S. N. Ojha, and H. S. Takhar, "Propagation of a shock wave in a rotating interplanetary atmosphere with increasing energy," Theor. Chim. Acta 44(1), 87–98 (1999).
[3]. J. P. Vishwakarma and G. Nath, "Propagation of a cylindrical shock wave in a rotating dusty gas with heat conduction and radiation heat flux," Phys. Scr. 81(4), 045401 (2010).
[4]. J. P. Vishwakarma and G. Nath, "Cylindrical shock wave generated by a piston moving in a non-uniform self-gravitating rotational axisymmetric gas in the presence of conduction and radiation heat- flux," Adv. Eng. Res. 2, 537–576 (2011).
[5]. J. P. Vishwakarma and S. Vishwakarma, "Magnetogasdynamic cylindrical shock wave in a rotating gas with variable density," Int. J. Appl. Mech. Eng. 12(1), 283–297 (2007).
[6]. M. Hishida, T. Fujiwara, and P. Wolanski, "Fundamentals of rotating detonations," Shock Waves 19(1), 1–10 (2009).
[7]. V. A. Levin and G. A. Skopina, "Detonation wave propagation in rotational gas flows," J. Appl. Mech. Tech. Phys. 45(4), 457–460 (2004).
[8]. G. Nath, "Magnetogasdynamic shock wave generated by a moving piston in a rotational axisymmetric isothermal flow of perfect gas with variable density," Adv. Space Res. 47(9), 1463–1771 (2011).
[9]. N. Zhao, A. Mentrelli, T. Ruggeri, and M. Sugiyama, "Admissible shock waves and shock-induced phase transitions in a van der Waals fluid," Phys. Fluids 23(8), 086101 (2011).

ISSN No: 2456-2165 https://doi.org/10.38124/ijisrt/25jun1266

[10]. G. Nath, “Self-similar solution of cylindrical shock wave propagation in a rotational axisymmetric mixture of a non-ideal gas and small solid particles,” Meccanica 47, 1797–1814 (2012).
[11]. S. Yadav, D. Singh, and R. Arora, “Lie group of invariance technique for analyzing propagation of strong shock wave in a rotating non-ideal gas with azimuthal magnetic field,” Math. Methods Appl. Sci. (published online 2022).
[12]. G. Nath, “Propagation of a cylindrical shock wave in a rotational axisymmetric isothermal flow of a non-ideal gas in magnetogasdynamics,” Math. Methods Appl. Sci. Ain Shams Eng. J. 3(4), 393–401 (2012).
[13]. M. Devi, D. Singh, and R. Arora, “Similarity solutions for cylindrical shock waves in a non-ideal gas under the action of monochromatic radiation,” J. Phys. A 54, 125701 (2021).
[14]. G. Nath, “Propagation of exponential shock wave in an axisymmetric rotating non-ideal dusty gas,” Indian J. Phys. 90(9), 1055–1068 (2016).
[15]. J. P. Vishwakarma, A. K. Maurya, and K. K. Singh, “Self-similar adiabatic flow headed by a magnetogasdynamic cylindrical shock wave in a rotating non-ideal gas,” Geophys. Astrophys. Fluid Dyn. 101(2), 155–168 (2007).
[16]. D. I. Pullin, W. Mostert, V. Wheatley, and R. Samtaney, “Converging cylindrical shocks in ideal magnetohydrodynamics,” Phys. Fluids 26(9), 097103 (2014).
[17]. D. Singh and R. Arora, “Similarity solutions for imploding shocks in a nonideal magnetoga sdynamics,” Int. J. Appl. Comput. Math. 6(2), 46 (2020).
[18]. R. Arora, “Spherical shock waves in magneto-gas-dynamics,” Can. Appl. Math. Q. 15(1), 1–12 (2007).
[19]. D. Singh and R. Arora, “Propagation of shock waves in a non-ideal gas under the action of magnetic field,” Math. Methods Appl. Sci. 44, 1514 (2021).
[20]. D. Singh, R. Arora, and A. Chauhan, “Similarity solutions for strong shock waves in magnetogasdynamics under a gravitational field,” Ric. Mat. (published online 2022).
[21]. C. Radha and V. D. Sharma, “Imploding cylindrical shock in a perfectly conducting and radiating gas,” Phys. Fluids B 5(12), 4287– 4294 (1993).
[22]. J. K. Hunter and G. Ali, “Wave interactions in magnetohydrodynamics,” Wave Motion 27(3), 257–277 (1998).
[23]. P. A. Carrus, P. A. Fox, F. Haas, and Z. Kopal, “The propagation of shock waves in a stellar model with continuous density distribution,” Astrophysi. J. 113, 496 (1951).
[24]. J. P. Vishwakarma and G. Nath, “Spherical shock wave generated by a moving piston in mixture of a non-ideal gas and small solid particles under a gravitational field,” Commun. Nonlinear Sci. Numer. Simul. 17(6), 2382–2393 (2012).
[25]. G. Nath, “Self-similar flow behind a spherical shock wave in a non- ideal dusty gas under a gravitational field: Isothermal flow,” Adv. Space Res. 52(7), 1304–1313 (2013).
[26]. G. Nath, “Unsteady isothermal flow behind a magnetogasdynamic shock wave in a self-gravitating gas with exponentially varying density,” J. Theor. Appl. Phys. 8(3), 131 (2014).
[27]. G. W. Bluman and J. D. Cole, Similarity Methods for Differential Equations (Springer, Berlin, 1974).
[28]. G. W. Bluman and S. Kumei, Symmetries and Differential Equations (Springer, New York, 1989).
[29]. P. E. Hydon, Symmetry Methods for Differential Equations: A Beginners Guide (Cambridge University Press, London, 2000).
[30]. J. D. Logan and J. D. J. Perez, “Similarity solutions for reactive shock hydrodynamics,” SIAM J. Appl. Math. 39(3), 512–527 (1980).
[31]. F. Oliveri and M. P. Speciale, “Exact solutions to the ideal magneto- gas-dynamics equations through Lie group analysis and substitution principles,” J. Phys. A 38, 8803 (2005).
[32]. S. Kumar, W.-X. Ma, and A. Kumar, “Lie symmetries, optimal system and group-invariant solutions of the (3 þ 1)-dimensional generalized KP equation,” Chin. J. Phys. 69, 1–23 (2021).
[33]. H. Sharma and R. Arora, “Similarity solutions of cylindrical shock waves in non-ideal magnetogasdynamics with thermal radiation,” Differ. Equations Dyn. Syst. 27(13), 169–180 (2019).
[34]. J. Jena, “Self-similar solutions in a plasma with axial magnetic field (h-pinch),” Meccanica 47(5), 1209–1215 (2012).
[35]. S. Chauhan, A. Chauhan, and R. Arora, “Similarity solutions of converging shock waves in an ideal relaxing gas with dust particles,” Eur. Phys. J. Plus 135, 825 (2020).
[36]. G. Nath, “Propagation of a strong cylindrical shock wave in a rotational axisymmetric dusty gas with exponentially varying density,” Res. Astron. Astrophys. 10(5), 445 (2010).
[37]. S. Kumar, “Some new families of exact solitary wave solutions of the Klein– Gordon–Zakharov equations in plasma physics,” Pramana 95(4), 161 (2021).
[38]. S. Kumar, K. S. Nisar, and A. Kumar, “A (2 þ 1)-dimensional generalized Hirota–Satsuma–Ito equations: Lie symmetry analysis, invariant solutions and dynamics of soliton solutions,” Results Phys. 28, 104621 (2021).
[39]. S. Kumar and D. Kumar, “Generalised exponential rational function method for obtaining numerous exact soliton solutions to a (3 þ 1)- dimensional Jimbo–Miwa equation,” Pramana 95(4), 152 (2021).
[40]. D. D. Laumbach and R. F. Probstein, “Self-similar strong shocks with radiation in a decreasing exponential atmosphere,” Phys. Fluids 13(5), 1178–1183 (1970).
[41]. I. Lerche, “Mathematical theory of cylindrical isothermal blast waves in a magnetic field,” Aust. J. Phys. 34(3), 279–302 (1981).
[42]. P. L. Sachdev and S. Ashraf, “Converging spherical and cylindrical shocks with zero temperature gradient in the rear flow field,” Z. Ang. Math. Phys. 22(6), 1095–1102 (1971).
[43]. G. Nath and A. Devi, “A self-similar solution for

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unsteady adiabatic and isothermal flows behind the shock wave in a non-ideal gas using Lie group analysis method with azimuthal or axial magnetic field in rotating medium," Eur. Phys. J. Plus 136(5), 477 (2021).
[44]. G. Nath and S. Singh, "Similarity solutions for magnetogasdynamic cylindrical shock wave in rotating ideal gas using Lie group theoretic method: Isothermal flow," Int. J. Geometr. Methods Mod. Phys. 17, 2050123 (2020).
[45]. G. Nath, "Propagation of shock wave in a rotational axisymmetric ideal gas with density varying exponentially and azimuthal magnetic field: Isothermal flow," Indian J. Phys. 95, 163–175 (2021).
[46]. P. Rosenau and S. Frankenthal, "Equatorial propagation of axisymmetric magnetohydrodynamic shocks," Phys. Fluids 19(12), 1889–1899 (1976).